CHAPTER - 5

MAGNETOSTATICS

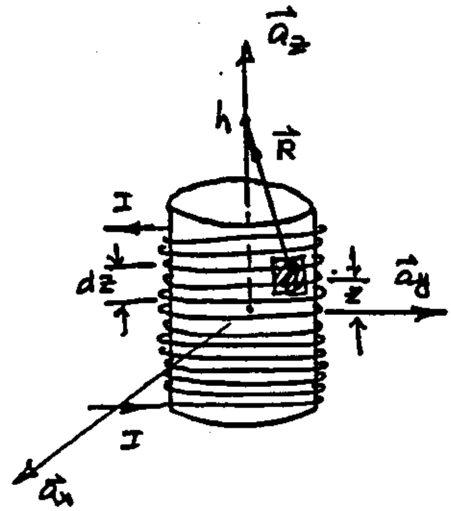
Exercise 5.1 From example-5.1, for $a=0$ and $b=\infty$, we have

$$\vec{B} = \frac{\mu_0 I}{4\pi\rho}\,\vec{a}_\phi$$

Exercise 5.2 From example 5.1, for $a=-L$ and $b=L$, we get

$$\vec{B} = \frac{\mu_0 I L}{2\pi\rho}\,\frac{1}{\sqrt{\rho^2+L^2}}\,\vec{a}_\phi$$

Exercise 5.3



Let $n=\frac{N}{L}$, $\vec{dl} = dz\,\vec{a}_z$, $\vec{J}_s = nI\,\vec{a}_\phi$

$ds = b\,dz\,d\phi$ $\qquad \vec{R} = -b\vec{a}_\rho + (h-z)\vec{a}_z$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{J}_s\times\vec{R}}{R^3}\,ds$$

$$= \frac{\mu_0}{4\pi}\int_{-\frac{L}{2}}^{L/2}\frac{nIb^2\,dz}{\left[b^2+(z-h)^2\right]^{3/2}}\,\vec{a}_z\int_0^{2\pi} d\phi$$

$$+\ \frac{\mu_0}{4\pi}\int_{-\frac{L}{2}}^{L/2}\frac{(h-z)\,nIb\,dz}{\left[b^2+(z-h)^2\right]^{3/2}}\cancel{\int_0^{2\pi}\vec{a}_\rho\,d\phi}^{\,0}$$

Let $z-h = b\tan\theta$, $dz = b\sec^2\theta\,d\theta$

$$\int\frac{dz}{\left[b^2+(z-h)^2\right]^{3/2}} = \frac{1}{b^2}\int\cos\theta\,d\theta$$

$$= \frac{1}{b^2}\,\frac{z-h}{\sqrt{b^2+(z-h)^2}}$$

Thus, $$B_z = \frac{\mu_0 nI}{2}\left[\frac{\frac{L}{2}-h}{\sqrt{b^2+(\frac{L}{2}-h)^2}} + \frac{\frac{L}{2}+h}{\sqrt{b^2+(\frac{L}{2}+h)^2}}\right],$$

When $h=-\frac{L}{2}$,

$$B_z = \frac{\mu_0 nI}{2}\cdot\frac{L}{\sqrt{b^2+L^2}}$$

When $h=L/2$,

$$B_z = \frac{\mu_0 nI}{2}\cdot\frac{L}{\sqrt{b^2+L^2}}$$

At center, $h=0$,

$$B_z = \frac{\mu_0 nI}{2}\,\frac{L}{\sqrt{b^2+L^2/4}}$$

Finally, when $L\to\infty$

$$B_z = \mu_0 nI = \mu_0\frac{NI}{L}$$

Exercise 5.4 Since $q_1\vec{u}_1 = I_1 d\vec{l}_1$, then $\vec{F}_1 = \int_{c_1} I_1 d\vec{l}_1 \times \vec{B}_2$

However, $\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{q_2 \vec{u}_2 \times \vec{R}}{R^3}$. Substitute $I_2 d\vec{l}_2 = q_2 \vec{u}_2$ and get

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \int \frac{I_2 d\vec{l}_2 \times \vec{R}}{R^3} \quad .$$

Exercise 5.5 $\int_{-L}^{L} dz_1 \int_{-a}^{a} \frac{b\vec{a}_y + (z_2 - z_1)\vec{a}_z}{[b^2 + (z_2-z_1)^2]^{3/2}} dz_2 = ?$

Verify (5.17)

y-component: $\int_{-L}^{L} dz_1 \int_{-a}^{a} \frac{b\,dz_2}{[b^2+(z_2-z_1)^2]^{3/2}}$ Let $z_2 - z_1 = b\tan\theta$, $dz_2 = b\sec^2\theta\, d\theta$

Then $\int \frac{b\,dz_2}{[b^2+(z_2-z_1)^2]^{3/2}} = \frac{1}{b}\int \cos\theta\, d\theta = \frac{1}{b}\sin\theta = \frac{1}{b} \frac{z_2 - z_1}{\sqrt{b^2+(z_2-z_1)^2}}$

Thus, $\int_{-L}^{L} dz_1 \int_{-a}^{a} \frac{b\,dz_2}{[b^2+(z_2-z_1)^2]^{3/2}} = \frac{1}{b}\int_{-L}^{L} \left[\frac{a+z_1}{\sqrt{b^2+(a+z_1)^2}} - \frac{a-z_1}{\sqrt{b^2+(a-z_1)^2}}\right]$

$$= \frac{2}{b}\left[\sqrt{(L+a)^2+b^2} - \sqrt{(L-a)^2+b^2}\right]$$

z-component:

$$\int_{-L}^{L} dz_1 \int_{-a}^{a} \frac{(z_2-z_1)\,dz_2}{[b^2+(z_2-z_1)^2]^{3/2}} = \int_{-L}^{L}\left(\frac{1}{\sqrt{b^2+(z_1+a)^2}} - \frac{1}{\sqrt{b^2+(z_1-a)^2}}\right)dz_1$$

$$= \ln\left[\frac{(z_1+a) + \sqrt{(z_1+a)^2+b^2}}{(z_1-a) + \sqrt{(z_1-a)^2+b^2}}\right]_{-L}^{L} = 0$$

Similarly, (5.18) can be verified.

Exercise 5.6 By direct substitution,

$$\vec{F} = -30.2\,\vec{a}_y \quad \mu N$$

Exercise 5.7 $m = NIA = 10 \times 15 \times 10 \times 20 \times 10^{-4} = 3$

$$|T| = |\vec{m} \times \vec{B}| = 3 \times 0.8 \sin\theta = 2.4 \sin\theta \quad N.m$$

Exercise 5.8 $m = NIA = 25 \times 4 \times 2.5 \times 10^{-4} I = 0.025I$

$$|T| = mB\sin\theta = 0.025I \times 0.2 \sin 90^\circ = 0.005I = k\theta$$

for $\theta = 1^\circ$ $\quad I = 10\ mA/deg$.

(c) Full-scale deflection: $I = 10 \frac{mA}{deg} . 50\,deg = 500\ mA$

(b) Per-scale division $I = 500 \times 10^{-3}/100 = 5\ mA/scale\text{-}div$

Exercise 5.9 $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \vec{a}_\phi$, $\nabla \cdot \vec{B} = \frac{1}{\rho} \frac{\partial B_\phi}{\partial \phi} = \frac{1}{\rho} \frac{\partial}{\partial \phi}\left(\frac{\mu_0 I}{2\pi\rho}\right) = 0$

Exercise 5.10 $\vec{B} = \left[-\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(b-y)}\right]\vec{a}_x$, $\vec{ds} = -dy\,dz\,\vec{a}_x$

$$\Phi = \int_s \vec{B} \cdot \vec{ds} = \frac{\mu_0 I}{2\pi} \int_a^{b-a} \frac{1}{y} dy \int_0^L dz + \frac{\mu_0 I}{2\pi} \int_a^{b-a} \frac{1}{y-b} dy \int_0^z dz$$

$$= \frac{\mu_0 IL}{2\pi} \ln\left(\frac{b-a}{a}\right) + \frac{\mu_0 IL}{2\pi} \ln\left(\frac{a}{b-a}\right) = 0$$

Exercise 5.11 $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \vec{a}_\phi$ $\quad \vec{ds} = dy\,dz\,\vec{a}_x$ $\quad \rho = \sqrt{x^2+y^2}$

$\vec{B} \cdot \vec{ds} = \frac{\mu_0 I}{2\pi\rho} dy\,dz\ \vec{a}_x \cdot \vec{a}_\phi$ $\qquad \vec{a}_x \cdot \vec{a}_\phi = -\sin\phi = \frac{-y}{\sqrt{x^2+y^2}}$ where $x = b$

$$\Phi = -\frac{\mu_0 I}{2\pi} \int_{-b}^{b} \frac{y}{y^2+b^2} dy \int_{-b}^{b} dz \Rightarrow -\Phi = \frac{1}{2} \cdot \frac{\mu_0 I}{2\pi} \ln(y^2+b^2)\Big|_{-b}^{b} (2b) = 0$$

Exercise 5.12 $\vec{B} = 12x\,\vec{a}_x + 25y\,\vec{a}_y + cz\,\vec{a}_z$

$\nabla \cdot \vec{B} = 12 + 25 + c$. Since $\nabla \cdot \vec{B}$ must be zero, $c = -37$

Exercise 5.13 $\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\ \vec{a}_r$, $\quad \vec{B} = B\vec{a}_z$, $\quad \vec{a}_r \cdot \vec{a}_z = \cos\theta$

$$\underset{(at\ r=R)}{\Phi} = B R^2 \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = BR^2 (2\pi) \frac{1}{2} \sin^2\theta \Big|_0^{\pi/2} = \pi B R^2$$

Exercise 5.14 $\quad \vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{a}_\phi \qquad \vec{ds} = d\rho\, dz\, \vec{a}_\phi \qquad I = 80A$

$$\Phi = \int_s \vec{B}\cdot\vec{ds} = \frac{\mu_0 I}{2\pi}\int_{0.01}^{0.1}\frac{1}{\rho}d\rho\int_0^{100}dz = \frac{100\mu_0 I}{2\pi}\ln(10) = 3.68 \text{ mWb}$$

Exercise 5.15

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2}\frac{dz}{R}\vec{a}_z = \frac{\mu_0 I}{4\pi R}L\vec{a}_z \text{ when } R \gg L. \text{ (Note: } R \cong r)$$

$$= \frac{\mu_0 I}{4\pi R}L\cos\theta\,\vec{a}_r = -\frac{\mu_0 IL}{4\pi R}\sin\theta\,\vec{a}_\theta$$

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{\mu_0 I}{4\pi r}L\cos\theta & -\frac{\mu_0 IL}{4\pi}\sin\theta & 0 \end{vmatrix}\frac{1}{r^2\sin\theta} = \frac{\mu_0 IL}{4\pi r^2}\sin\theta\,\vec{a}_\phi$$

Exercise 5.16 $\quad \rho \le 10$ cm $\qquad \vec{J} = \frac{100}{\pi a^2}\vec{a}_z$, $a = 10$ cm

$$I_{enc} = \frac{100}{\pi a^2}\pi\rho^2 = 100\frac{\rho^2}{a^2}, \quad \oint_c \vec{H}\cdot\vec{dl} = I_{enc} \Rightarrow \vec{H} = \frac{100\rho}{2\pi a^2}\vec{a}_\phi = 1591.55\rho\,\vec{a}_\phi \text{ A/m}$$

$$\rho \ge a, \quad I_{enc} = 100 \text{ A}, \quad \vec{H} = \frac{100}{2\pi\rho}\vec{a}_\phi = \frac{15.915}{\rho}\vec{a}_\phi \text{ A/m}$$

Exercise 5.17 $\quad N = 500$ Turns, $a = 15$ cm, $b = 20$ cm, $h = 5$ cm, $I = 2A$

$$H_\phi = \frac{NI}{2\pi\rho} = \frac{159.15}{\rho} \text{ A/m}, \quad B_\phi = \mu_0 H_\phi = 4\pi\times10^{-7}\times\frac{159.15}{\rho} = \frac{200\times10^{-6}}{\rho} \text{ T}$$

$$\Phi = \frac{\mu_0 NI}{2\pi}h\ln(b/a) = \frac{4\pi\times10^{-7}\times500\times2\times0.05}{2\pi}\ln\left(\frac{20}{15}\right) = 2.88\ \mu\text{Wb}$$

At the mean radius: $\rho = (20+15)/2 = 17.5$ cm, $H_\phi = 909.43$ A/m

$B_\phi = \mu_0 H_\phi = 1.143$ mT, $\quad \Phi = 1.143\times10^{-3}\times(0.2-0.15)(0.05) = 2.86\ \mu$Wb

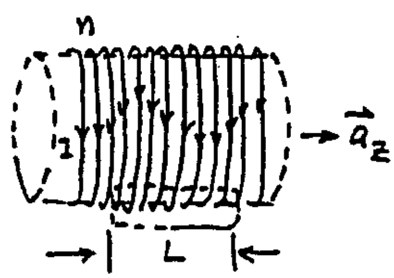
Percent error: $\frac{2.88-2.86}{2.88} = 0.7\%$

Exercise 5.18

$\oint_c \vec{H}\cdot\vec{dl} = H_z L$

$I_{enc} = nIL$, Thus $H_z = nI$

$B_z = \mu_0 nI, \quad \Phi = \mu_0 nI\pi b^2$



Exercise 5.19 $H_z = nI$ $\quad B_z = \mu_o \mu_r nI$

$$M_z = \chi_m H_z = (\mu_r - 1)nI, \quad \vec{J}_{vb} = \nabla \times \vec{M} = 0$$

$$\vec{J}_{sb}\Big|_{\rho=b} = \vec{M} \times \vec{a}_\rho = (\mu_r - 1)nI(\vec{a}_z \times \vec{a}_\rho) = (\mu_r - 1)nI\,\vec{a}_\phi$$

Exercise 5.20 $\mu_r = 1200$ $\quad N = 500$ turns, $I = 2A$, $a = 0.15$ m $\quad b = 0.2$m $\quad h = 0.05$m

$$M_\phi = \frac{1199 \times 500 \times 2}{2\pi\rho} = \frac{190.83}{\rho}\vec{a}_\phi \qquad \vec{J}_{vb} = 0$$

$$\vec{B} = \mu_o \mu_r \vec{H} = \mu_o \mu_r \frac{NI}{2\pi\rho} = \frac{0.24}{\rho}\vec{a}_\phi, \quad \Phi = \int_{0.15}^{0.2} \frac{0.24}{\rho} d\rho \int_0^{0.05} dz = 3.45 \text{ mWb}$$

$$\vec{J}_{sb}\Big|_{\text{Top surface}} = \frac{190.83}{\rho}\vec{a}_\rho \qquad \vec{J}_{sb}\Big|_{\text{Bottom surface}} = -\frac{190.83}{\rho}\vec{a}_\rho$$

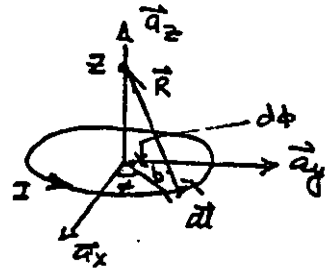
$$\vec{J}_{sb}\Big|_{\rho=a} = \frac{190.83}{0.15}\vec{a}_z = 1272.2\,\vec{a}_z, \quad J_{sb}\Big|_{\rho=b} = -\frac{190.83}{0.2}\vec{a}_z = -954.15\,\vec{a}_z$$

Exercise 5.21 $\vec{H} = \frac{NI}{2\pi\rho}\vec{a}_\phi$. mmf drop from $P_1(\rho_1, \phi_1, 0)$ to $P_2(\rho_2, \phi_2, 0)$ is

$$\int_C \vec{H}\cdot d\vec{l} = \frac{NI}{2\pi}\int_{\phi_1}^{\phi_2} d\phi = \frac{NI}{2\pi}(\phi_2 - \phi_1)$$

Exercise 5.22 Since $\vec{H} = \frac{\vec{B}}{\mu}$, For a given flux density, $\vec{H}$ is inversely proportional to μ. As μ goes up, $\vec{H}$ goes down.

Exercise 5.23



$$\vec{H} = \int_C \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3} \Rightarrow$$

$$H_z = \frac{Ib^2}{4\pi}\int_0^{2\pi} \frac{d\phi}{(b^2+z^2)^{3/2}} = \frac{Ib^2}{2(b^2+z^2)^{3/2}}$$

$$\mathcal{F} = \int_{z_1}^{z_2} H_z\,dz = \frac{I}{2}\left[\frac{z_2}{\sqrt{b^2+z_2^2}} - \frac{z_1}{\sqrt{b^2+z_1^2}}\right]$$

$$d\vec{l} = b\,d\phi\,\vec{a}_\phi$$

$$\vec{R} = -b\,\vec{a}_\rho + z\,\vec{a}_z$$

$$d\vec{l} \times \vec{R} = b^2 d\phi\,\vec{a}_z + bz\,d\phi\,\vec{a}_\rho$$

Due to symmetry, H_ρ will be zero.

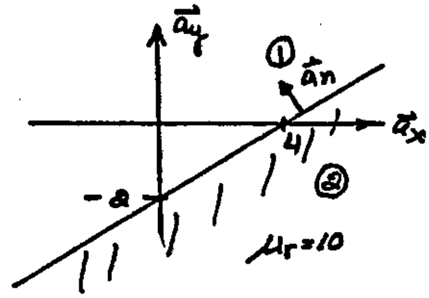
When $z_1 = 0$ and $z_2 = \infty$, $\mathcal{F} = \frac{I}{2}$.

Exercise 5.24

Plane: $2y - x + 4 \le 0$ $\quad 2y \le x - 4$

when $x = 0$, $y \le -2$. $\quad y \le \frac{x}{2} - 2$

when $y = 0$, $x \ge 4$



Let $f = 2y - x + 4$, $\nabla f = 2\vec{a}_y - \vec{a}_x$

$$\vec{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{2}{\sqrt{5}}\vec{a}_y - \frac{1}{\sqrt{5}}\vec{a}_x \qquad \vec{B}_2 = 2\vec{a}_x + 3\vec{a}_y - 5\vec{a}_z$$

Let $\vec{B}_1 = c_1\vec{a}_x + c_2\vec{a}_y + c_3\vec{a}_z$

$$\vec{a}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0 \Rightarrow -c_1 + 2c_2 = 4 \quad ①$$

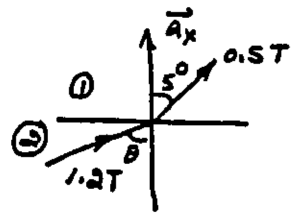
$$\vec{a}_n \times \left(\frac{\vec{B}_1}{\mu_0} - \frac{\vec{B}_2}{\mu_0\mu_r}\right) = 0 \Rightarrow \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ c_1 - 0.2 & c_2 - 0.3 & c_3 + 0.5 \end{vmatrix} = 0$$

Thus, $c_3 = -0.5$

$2c_1 + c_2 = 0.7$ ② $\quad$ From ① and ②: $c_1 = -0.52$ and $c_2 = 1.74$

$$\vec{B}_1 = -0.52\vec{a}_x + 1.74\vec{a}_y - 0.5\vec{a}_z$$

Exercise 5.25



$$B_{n1} = B_{n2} \Rightarrow \theta = \cos^{-1}\left[\frac{0.5\cos 5^\circ}{1.2}\right] = 65.48^\circ$$

$$H_{t1} = H_{t2} \Rightarrow \mu_r = \frac{1.2\sin 65.48^\circ}{0.5\sin 5^\circ} \cong 25$$

Exercise 5.26

$a = 0.1\,m$, $b = 0.14\,m$, $h = 0.04\,m$, $I = 0.5A$, $\mu_r = 500$

$H_\phi = 79.578\ A/m$, $B_\phi = \mu_0\mu_r H_\phi = 4\pi\times10^{-7}\times 500\times 79.578 \cong 50\ mT$

$\Phi = BA = 50\times10^{-3}\times(0.14-0.1)(0.04) = 80\ \mu Wb$

Mean radius: $\rho_m = (0.1+0.14)/2 = 0.12\,m$, $N = \frac{H(2\pi\rho_m)}{I} = 120$ Turns

$$\lambda = N\Phi \Rightarrow L = \frac{N\Phi}{I} = \frac{120\times 80\times10^{-6}}{0.5} = 19.2\ mH$$

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\times 19.2\times10^{-3}\times 0.5^2 = 2.4\ mJ$$

Energy density: $w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\times 50\times10^{-3}\times 79.578 = 1.99\ J/m^3$

$$W_m = \int_v w_m\,dv = 1.99\times(0.14-0.1)(0.04)2\pi\times 0.12 = 2.4\,mJ$$

Exercise 5.27 $\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$, $\vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{a}_\phi$ $\quad a \le \rho \le b$

$$\Phi = \int_s \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int_a^b \frac{1}{\rho}\,d\rho \int_0^1 dz = \frac{\mu_0 I}{2\pi}\ln(b/a)$$

Since $N=1$, $\lambda = \Phi \Rightarrow L = \frac{\lambda}{I} = \frac{\mu_0}{2\pi}\ln(b/a)$ H/m

$W_m = \frac{1}{2}LI^2 = \frac{\mu_0 I^2}{4\pi}\ln(b/a)$ J/m.

Also $w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{\mu_0 I^2}{8\pi^2\rho^2}$ and $W_m = \frac{\mu_0 I^2}{8\pi^2}\int_a^b \frac{1}{\rho^2}\rho\,d\rho \int_0^{2\pi} d\phi\,(1) = \frac{\mu_0 I^2}{4\pi}\ln(b/a)$

Exercise 5.28 For $\rho \le a$, $\vec{H} = \frac{I\rho}{2\pi a^2}\vec{a}_\phi$, $\vec{B} = \frac{\mu_0 I\rho}{2\pi a^2}\vec{a}_\phi$

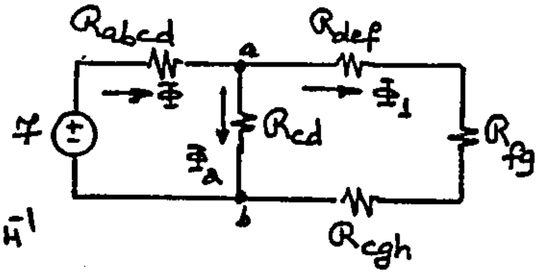
$$W_{m1} = \frac{1}{2}\int_v \vec{B}\cdot\vec{H}\,dv = \frac{\mu_0 I^2}{8\pi^2 a^4}\int_0^a \rho^3\,d\rho \int_0^{2\pi} d\phi\,(1) = \frac{\mu_0 I^2}{16\pi}$$

For $a \le \rho \le b$, the energy per unit length from Exercise 5.27 is

$$W_{m2} = \frac{\mu_0 I^2}{4\pi}\ln(b/a)$$

Thus, $W_m = W_{m1} + W_{m2} = \frac{\mu_0 I^2}{16\pi} + \frac{\mu_0 I^2}{4\pi}\ln(b/a)$

Exercise 5.29



$$\mathcal{R}_{fg} = \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times24\times10^{-4}} = 1.658\times10^{+6}\ H^{-1}$$

$$\mathcal{R}_{def} = \frac{28\times10^{-2}}{4\pi\times10^{-7}\times500\times24\times10^{-4}} = 185.68\times10^{3}\ H^{-1}$$

$$\mathcal{R}_{cd} = \frac{16\times10^{-2}}{4\pi\times10^{-7}\times500\times36\times10^{-4}} = 70.74\times10^{3}\ H^{-1}$$

$$\mathcal{R}_{cgh} = \frac{31.5\times10^{-2}}{4\pi\times10^{-7}\times500\times24\times10^{-4}} = 208.89\times10^{3}\ H^{-1}$$

$$\mathcal{R}_{abcd} = \frac{52\times10^{-2}}{4\pi\times10^{-7}\times500\times24\times10^{-4}} = 517.25\times10^{3}\ H^{-1}$$

$$\mathcal{R}_T = \mathcal{R}_{abcd} + (\mathcal{R}_{def} + \mathcal{R}_{fg} + \mathcal{R}_{cgh}) \parallel \mathcal{R}_{cd}$$
$$= 585.61\times10^{3}\ H^{-1}$$

$\Phi_1 = 0.05\times24\times10^{-4} = 120\ \mu Wb$

$\mathcal{F}_{ab} = \Phi_1(\mathcal{R}_{def} + \mathcal{R}_{fg} + \mathcal{R}_{cgh}) \Rightarrow$

$\Phi_2 = \frac{\mathcal{F}_{ab}}{\mathcal{R}_{cd}} = 3.483$ mWb

$\Phi = \Phi_1 + \Phi_2 = 3.603$ mWb

$\mathcal{F} = \Phi\mathcal{R}_T = 2109.73$

Hence; $I = \frac{\mathcal{F}}{1000} \simeq 2.11$ A

Exercise 5.30 Mean Radius, $R_m = 12.5\ cm$, $l_m = 2\pi R_m = 0.785\ m$

Area, $A = \frac{\pi}{4} \times (5 \times 10^{-2})^2 = \frac{78.54 \times 10^{-4}}{4}\ m^2$ $\qquad \Phi = 10\ mWb$

$$\mathcal{R} = \frac{0.785 \times 4}{4\pi \times 10^{-7} \times 1200 \times 78.54 \times 10^{-4}} = 265.26 \times 10^{3}\ H^{-1} \Rightarrow \mathcal{F} = \Phi \mathcal{R} = 2652.6\ At.$$

Exercise 5.31 The flux in the outer legs: $\Phi_a = \Phi_b = \frac{1}{2}\Phi_c = 3.5\ mWb$

$$B_a = B_b = \frac{3.5 \times 10^{-3}}{50 \times 10^{-4}} = 0.7\ T \quad \text{and} \quad B_c = \frac{7 \times 10^{-3}}{50 \times 10^{-4}} = 1.4\ T$$

From Fig. 5.37, $H_a \cong H_b \cong 620$ and $H_c \cong 1775$

$$\mathcal{F} = [1775(50-5) + 620(70+45)]10^{-2} \cong 1512\ At \Rightarrow I = \frac{1512}{500} = 3.02\ A$$

Exercise 5.32 Iteration-I : Applied mmf $= 500 \times 2 = 1000$ At

Region	Φ	A	B	H	l	mmf	
Leg c	66×10^{-4}	50×10^{-4}	1.32	1656	0.45	700	(Assume)
Leg a	33×10^{-4}	50×10^{-4}	0.66	550	1.15	632	
				Total		1332	(Too High)

Iteration-II

Leg	Φ	A	B	H	l	mmf
c	2.4×10^{-3}	50×10^{-4}	1.2	1228	0.45	550
a	1.2×10^{-3}	50×10^{-4}	0.6	530	1.15	610
						1160 (still High)

Iteration-III

leg	Φ	A	B	H	l	mmf
c	2.28×10^{-3}	50×10^{-4}	1.14	1067	0.45	480
a	1.14×10^{-3}	50×10^{-4}	0.57	520	1.15	598
						1078

(iterate one more time)

Iteration-IV

c	2.16×10^{-3}	50×10^{-4}	1.08	933	0.45	420	Error $\leq 2\%$
a	1.08×10^{-3}	"	0.54	515	1.15	598	
						1018	(ok)

Problem 5.1 From Eq(5.8), $\vec{B} = \frac{\mu_0 I}{2b}\vec{a}_z = \frac{4\pi\times10^{-7}\times10}{2\times2\times10^{-2}}\vec{a}_z = 314.16\,\vec{a}_z \ \mu T$

From (5.7), $\vec{B} = \frac{4\pi\times10^{-7}\times10\times4\times10^{-4}\vec{a}_z}{2(4\times10^{-4}+100\times10^{-4})^{3/2}} = 2.37\,\vec{a}_z \ \mu T$

c) $z = 10\,m$ $\quad \vec{B} = \frac{4\pi\times10^{-7}\times10\times4\times10^{-4}}{2(4\times10^{-4}+100)^{3/2}}\vec{a}_z = 2.51\,\vec{a}_z \ pT$

$$\vec{m} = IA\,\vec{a}_z = 10\times\pi\times4\times10^{-4}\,\vec{a}_z = 12.57\times10^{-3}\,\vec{a}_z$$

Problem 5.2 $b = 2\times10^{-3}\,m$ $\quad L = 1.2\times10^{-2}\,m$ $\quad n = 200$ $\quad I = 12\,A$

From Exercise 5.3, $\vec{B}_{center} = \frac{\mu_0 n I}{2}\frac{L}{\sqrt{(b^2+\frac{L^2}{4})}}\vec{a}_z = 2.86\,\vec{a}_z \ mT$

At the end: $\vec{B}_{end} = \frac{\mu_0 n I}{2}\frac{L}{\sqrt{b^2+L^2}}\vec{a}_z = 1.487\,\vec{a}_z \ mT$

Problem 5.3 $n = 2$ turns/mm $= 2000$ Turns/m, $B_z = 0.5T$

From Exercise 5.3, $B_z = \mu_0 n I \Rightarrow I = \frac{0.5}{4\pi\times10^{-7}\times2000} = 198.94\,A$

Problem 5.4 From Example 5.1, the $\vec{B}$ field in the mid-plane of a wire of length L is

$$B_{z1} = \frac{\mu_0 I}{2\pi(\frac{L}{2})}\cdot\frac{L/2}{\sqrt{(\frac{L}{2})^2+(\frac{L}{2})^2}} = \frac{1}{\sqrt{2}}\frac{\mu_0 I}{\pi L} \qquad \rho = \frac{L}{2}$$

For a square loop, the $\vec{B}$ field at the center will be

$$B_z = 4B_{z1} = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$$

When $L = 10\,cm$, $I = 120\,A$, then $B_z = 1.358\ mT$

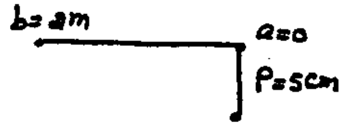
Since the coil has 500 turns, the total $\vec{B}$ field at the center of the loop is $\vec{B} = 500\,B_z\,\vec{a}_z \cong 0.68\,\vec{a}_z \ T$

Problem 5.5

a) Semi-circular section: $\vec R = -b\,\vec a_\rho$, $d\vec l = b\,d\phi\,\vec a_\phi$

$$\vec B = \frac{\mu_0 I}{4\pi}\int_c \frac{d\vec l \times \vec R}{R^3} = \frac{\mu_0 I}{4\pi b}\int_0^{\pi} d\phi\,\vec a_z = \frac{\mu_0 I}{4b}\vec a_z = 0.628\,\vec a_z \text{ mT}$$

b) Horizontal conductor:



From Example 5.1, $\vec B = \frac{\mu_0 I}{4\pi\rho}\frac{b}{\sqrt{\rho^2+b^2}}$

$= 0.2\,\vec a_z$ mT

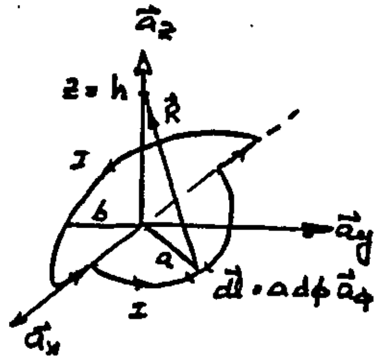
c) From the other conductor: $\vec B = 0.2\,\vec a_z$ mT

d) From Example 5.1, the $\vec B$ field in the mid-plane

is $$\vec B = \frac{\mu_0 I}{2\pi\rho}\left[\frac{L/2}{\sqrt{\rho^2+(L/2)^2}}\right]\vec a_z = 250\,\vec a_z \text{ nT}$$ $L=10$ cm, $\rho = 2$ m

Thus: $\vec B = [0.628 + 0.2 + 0.2 + 250\times10^{-6}]\,10^{-3}\,\vec a_z \approx 1.028\,\vec a_z$ mT

Problem 5.6



Semi-circular loop of radius a

$$\vec B_a = \frac{\mu_0 I}{4\pi}\int_0^{\pi} \frac{a\,d\phi\,\vec a_\phi \times (-a\,\vec a_\rho + z\,\vec a_z)}{(a^2+z^2)^{3/2}}$$

$\vec R = -a\,\vec a_\rho + z\,\vec a_z$

$$= \frac{\mu_0 I}{4\pi\,(a^2+z^2)^{3/2}}\left[\vec a_z\, a^2\int_0^{\pi} d\phi + a z\int_0^{\pi}\vec a_\rho\, d\phi\right]$$

$\vec a_\rho = \vec a_x\cos\phi + \vec a_y\sin\phi$

$$= \frac{\mu_0 I a^2}{4(a^2+z^2)^{3/2}}\vec a_z - \frac{\mu_0 I a z}{2\pi\,(a^2+z^2)^{3/2}}\vec a_y$$

similarly for the other semi-circular loop, of radius b,

$$\vec B_b = \frac{\mu_0 I b^2}{4(b^2+z^2)^{3/2}}\vec a_z - \frac{\mu I b z}{2\pi(b^2+z^2)^{3/2}}\vec a_y$$

For the straight conductor from $x = b$ to $x = a$, $d\vec l = dx\,\vec a_x$, $\vec R = -x\,\vec a_x + z\,\vec a_z$

$d\vec l \times \vec R = -z\,dx\,\vec a_y$

$$\vec B_{ba} = -\vec a_y\,\frac{\mu_0 I z}{4\pi}\int_b^a \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi z}\left[\frac{b}{\sqrt{b^2+z^2}} - \frac{a}{\sqrt{a^2+z^2}}\right]\vec a_y$$

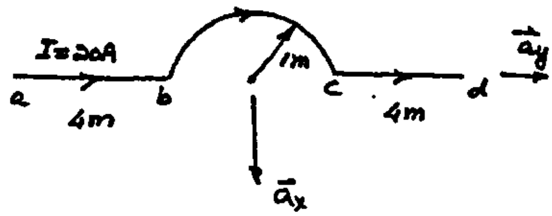
Contribution by the current-carrying conductor from $x=-a$ to $x=-b$ will also be the same. Thus,

$$\vec{B} = \vec{B}_a + \vec{B}_b + 2\vec{B}_{ba}$$

$$= \vec{a}_y\left[-\frac{\mu_0 I z}{2\pi}\left\{\frac{a}{\sqrt{a^2+z^2}} + \frac{b}{\sqrt{b^2+z^2}}\right\} + \frac{\mu_0 I}{2\pi z}\left\{\frac{b}{\sqrt{b^2+z^2}} - \frac{a}{\sqrt{a^2+z^2}}\right\}\right]$$

$$+ \vec{a}_z\left[\frac{\mu_0 I}{2\pi}\left(\frac{a^2}{(a^2+z^2)^{3/2}} + \frac{b^2}{(b^2+z^2)^{3/2}}\right)\right]$$

Problem 5.7 $\quad \vec{B} = 1.25\,\vec{a}_z\ T$



$$\vec{F}_{ab} = \int_a^b I\,d\vec{l}\times\vec{B} = 25\int_{-5}^{-1} dy\,\vec{a}_x = 100\,\vec{a}_x\ N$$

$$\vec{F}_{cd} = \int_c^d I\,d\vec{l}\times\vec{B} = 25\int_1^5 dy\,\vec{a}_x = 100\,\vec{a}_x\ N$$

$$\vec{F}_{bc} = 25\int_{3\pi/2}^{\pi/2}(\vec{a}_\phi\times\vec{a}_z)\,d\phi = 25\int_{3\pi/2}^{\pi/2}\vec{a}_\rho\,d\phi = 25\int_{3\pi/2}^{\pi/2}\cos\phi\,d\phi\,\vec{a}_x + \int_{3\pi/2}^{\pi/2}25\sin\phi\,d\phi\,\vec{a}_y$$

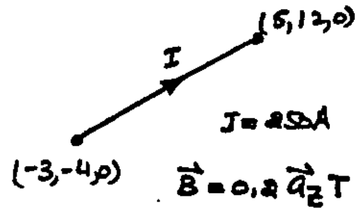
$$= 50\,\vec{a}_x\ N$$

$$\vec{F} = \vec{F}_{ab} + \vec{F}_{cd} + \vec{F}_{bc} = 250\,\vec{a}_x\ N$$

Problem 5.8 $\quad q = 500\,nC,\quad \vec{u} = 500\,\vec{a}_x + 2000\,\vec{a}_y\ m/s \quad \vec{B} = 1.2\,\vec{a}_z\ T$

$$\vec{F} = q(\vec{u}\times\vec{B}) = 500\times10^{-9}\left[500\,\vec{a}_x + 2000\,\vec{a}_y\right]\times 1.2\,\vec{a}_z = 1.2\,\vec{a}_x - 0.3\,\vec{a}_y\ mN$$

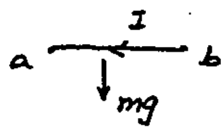
Problem 5.9 $\quad d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y$



$$d\vec{l}\times\vec{B} = -0.2\,dx\,\vec{a}_y + 0.2\,dy\,\vec{a}_x$$

$$\vec{F} = \int_c I\,d\vec{l}\times\vec{B} = -50\int_{-3}^{5} dx\,\vec{a}_y + 50\int_{-4}^{12} dy\,\vec{a}_x = 800\,\vec{a}_x - 400\,\vec{a}_y\ N$$

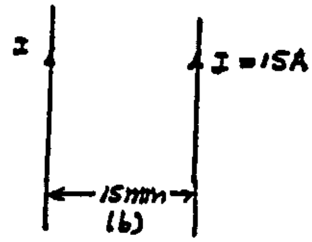
Problem 5.10 $\quad$ Current must be from b to a.



$$BIL = mg \Rightarrow I = \frac{mg}{BL} = \frac{0.5\times9.81}{0.9\times1.2} = 4.54\ A$$

Problem 5.11 From (5.15)

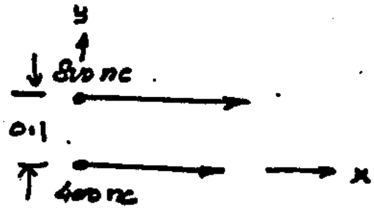
$$\vec{F} = -\frac{\mu_0}{2\pi b} I^2 L \, \vec{a}_\rho = -\frac{4\pi\times10^{-7}\times15^2\times0.5}{2\pi(15\times10^{-3})}\vec{a}_\rho$$



$= -1.5\,\vec{a}_\rho$ mN [−ve sign ⇒ Force of attraction]

Problem 5.12

$$Fe_{800} = \frac{800\times10^{-9}\times400\times10^{-9}\times9\times10^{9}}{(0.1)^2}\vec{a}_y$$



$= 0.288\,\vec{a}_y$ N

$$F_{m800} = \frac{4\pi\times10^{-7}}{4\pi(0.1)^2}\left[800\times10^{-9}\times400\times10^{-9}\left[20\times10^{6}\times50\times10^{6}\right]\vec{a}_x\times(\vec{a}_x\times\vec{a}_y)\right]$$

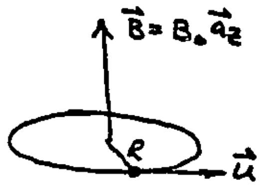
$= -3.2\,\vec{a}_y$ mN . Thus $\vec{F}_{TOT} = Fe_{800} + F_{m800} = 0.2848\,\vec{a}_y$ N

$$\left|\frac{F_e}{F_m}\right| = 90$$

Problem 5.13 $\vec{F}_{e800} = 0.288\,\vec{a}_y$ N exactly the same as in Problem 5.12

$\vec{F}_m = 0.0032\left[\vec{a}_x\times(-\vec{a}_x\times\vec{a}_y)\right] = 0.0032\,\vec{a}_y$ N

$\vec{F}_{TOT} = 0.2912\,\vec{a}_y$ N



Problem 5.14 $u_0 = 2.2\times10^{6}$ m/s, $R = 5.3\times10^{-11}$ m

Let $\vec{B} = B_0\vec{a}_z$ and $\vec{u} = u_0\vec{a}_\phi$

$$\vec{F}_e = -\frac{e^2}{4\pi\epsilon_0 R^2}\vec{a}_\rho = -\frac{(1.6\times10^{-19})^2\times9\times10^{9}}{(5.3\times10^{-11})^2}\vec{a}_\rho = -8.2\times10^{-8}\vec{a}_\rho$$

$$\vec{F}_m = -e u_0 B_0\vec{a}_\rho = -1.6\times10^{-19}\times2.2\times10^{6}B_0\vec{a}_\rho = -3.52\times10^{-13}B_0\vec{a}_\rho$$

Centripetal force: $\vec{F}_c = \dfrac{m_e u^2}{R}\vec{a}_\rho = \dfrac{9.1\times10^{-31}\times(2.2\times10^{6})^2}{5.3\times10^{-11}} = 8.31\times10^{-8}\vec{a}_\rho$

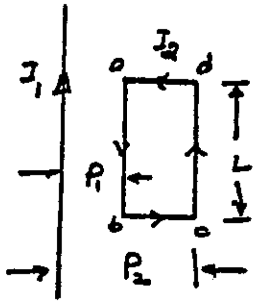
For $\vec{F}_e + \vec{F}_m + \vec{F}_c = 0 \Rightarrow B_0 = \dfrac{(8.31-8.2)10^{-8}}{3.52\times10^{-13}} \cong 3.1$ kT

Problem 5.15 $\vec{B} = \dfrac{\mu_0}{4\pi R^2}\left[q\vec{u}\times\vec{a}_r\right] = \mu_0\vec{u}\times\left[\dfrac{q\vec{a}_r}{4\pi R^2}\right] = \mu_0\epsilon_0\,\vec{u}\times\vec{E}$

Problem 5.16 $I_1 = 10\text{ A}$, $I_2 = 20\text{A}$ $b = 10\text{ cm}$

$$\vec{F} = \frac{\mu_o}{2\pi b} I_1 I_2 \,\vec{a}_\rho = 400\,\vec{a}_\rho \;\mu N$$

Problem 5.17 Force on bc is balanced by that on da

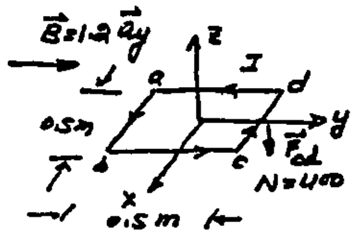


Thus: $\vec{F}_2 = \vec{F}_{ab} + \vec{F}_{cd} = \dfrac{\mu_o I_1 I_2}{2\pi}\left[\dfrac{L}{\rho_1} - \dfrac{L}{\rho_2}\right]\vec{a}_\rho$

When $I_1 = 500\text{ A}$, $I_2 = 20\text{ A}$, $L = 80\text{ cm}$, $\rho_1 = 20\text{ cm}$, $\rho_2 = 40\text{ cm}$,

$$\vec{F}_2 = 4\,\vec{a}_\rho \;mN$$

Problem 5.18 a) Parallel: $I = 8\text{A}$, $L = 0.5\text{m}$



$\vec{F}_{ab} = 400 I\, \vec{L} \times \vec{B} = 400 \times 8 \times 0.5 \times 1.2\,\vec{a}_z = 1920\,\vec{a}_z$ N

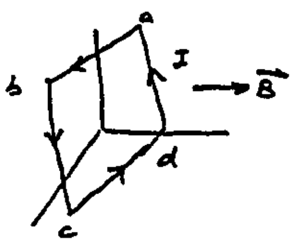
$F_{cd} = -1920\,\vec{a}_z$ $\quad \vec{F}_{bc} = 0$ $\quad \vec{F}_{ad} = 0$

$\vec{T} = \vec{r} \times \vec{F} = (0.5\vec{a}_y) \times \vec{F}_{cd} = -960\,\vec{a}_x$ N.m (Rotation clockwise)

Verify: $\vec{T} = \vec{m} \times \vec{B} = (400\, I A\, \vec{a}_z) \times (1.2\,\vec{a}_y) = 400 \times 8 \times (0.5)^2 \times 1.2\,(-\vec{a}_x)$

$= -960\,\vec{a}_x$ N.m.

b) Perpendicular:



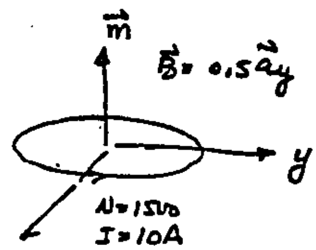
$\vec{F}_{ab} = 400 \times 8 \times 0.5[\vec{a}_x \times \vec{a}_y] = 1920\,\vec{a}_z$ N

$\vec{F}_{cd} = -1920\,\vec{a}_z$ N

$\vec{F}_{bc} = 400 \times 8 \times 0.5[-\vec{a}_z \times \vec{a}_y] = 1920\,\vec{a}_x$ N, $\vec{F}_{da} = -1920\,\vec{a}_x$ N

Forces cancel. No torque developed.

Problem 5.19 $\vec{m} = nIA\,\vec{a}_z = 1500 \times 10 \times (1.2 \times 10^{-2})^2 \pi$



$= 6.786\,\vec{a}_z$

$T = mB\sin\theta = k\theta \Rightarrow$

$$k = \frac{6.786 \times 0.5 \times \sin 30^\circ}{\pi/6} = 3.24 \text{ N·m/rad}$$

$r = 1.2\text{cm}$

Problem 5.20 $A = 100\ cm^2 \quad N=1200$ Turns $\quad I = 25A$

$$m = NIA = 1200\times 25\times 100\times 10^{-4} = 300 \qquad B=1.2T$$

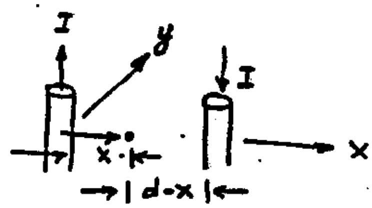
$$T = mB\sin\theta = 360 \sin\theta \qquad W = \int_0^{\pi} 360\sin\theta\, d\theta = 720J$$

Problem 5.21 $\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi \ , \ \vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{a}_\phi \qquad I = 100\ A$

$$\Phi = \int_S \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int_{0.01}^{0.1}\frac{1}{\rho}d\rho\int_{+0.05}^{0.5}dz = \frac{4\pi\times 10^{-7}}{2\pi}\times 100\times \ln\left(\frac{0.1}{0.01}\right)0.45 = 20.72\ \mu Wb$$

Problem 5.22

$$\vec{H} = \frac{I}{2\pi}\left[\frac{1}{x} + \frac{1}{d-x}\right]\vec{a}_y$$



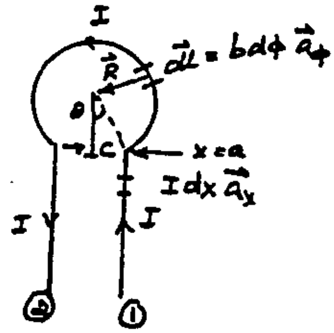
$$\Phi = \int_S \mu_0\vec{H}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int_{0.001}^{0.099}\frac{1}{x}dx\int_0^1 dz + \frac{\mu_0 I}{2\pi}\int_{0.099}^{0.001}\frac{1}{d-x}dx\int_0^1 dz$$

$$= \frac{4\pi\times 10^{-7}\times 200}{2\pi}\left[\ln\left(\frac{0.099}{0.001}\right) + \ln\left(\frac{0.099}{0.001}\right)\right] = 367.6\ \mu Wb$$

$I = 200A$

The flux would be zero if the currents were in the same direction

Problem 5.23



For the loop: $\vec{R} = -b\vec{a}_\rho \quad d\vec{l} = bd\phi\,\vec{a}_\phi,\ d\vec{l}\times\vec{R} = b^2 d\phi\,\vec{a}_z$

$$\vec{H}_0 = \frac{I}{4\pi}\int_C \frac{d\vec{l}\times\vec{R}}{R^3} = \vec{a}_z\ \frac{I}{4\pi b}\int_\theta^{2\pi-\theta} d\phi = \frac{I}{2\pi b}(\pi-\theta)\,\vec{a}_z$$

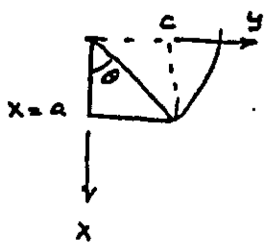
Long-lead ①: $\vec{R} = -x\,\vec{a}_x - c\,\vec{a}_y,\ d\vec{l}\times\vec{R} = -c\,dx\,\vec{a}_z$

$$\vec{H}_1 = -\vec{a}_z\ \frac{Ic}{4\pi}\int_\infty^a \frac{dx}{(x^2+c^2)^{3/2}} = -\vec{a}_z\left[\frac{x}{\sqrt{x^2+c^2}}\Big|_\infty^a\right]\frac{I}{4\pi c} = \vec{a}_z\left[1-\frac{a}{\sqrt{a^2+c^2}}\right]\frac{I}{4\pi c}$$

Similarly for lead ②: $\vec{H}_2 = \vec{a}_z\ \frac{I}{4\pi c}\left[1-\frac{a}{\sqrt{a^2+c^2}}\right] \Rightarrow \vec{H}_1+\vec{H}_2 = \frac{I}{2\pi c}\left[1-\frac{a}{\sqrt{a^2+c^2}}\right]\vec{a}_z$

Since $a^2+c^2 = b^2$ and $a = b\cos\theta,\ c = b\sin\theta$,

$$\vec{H}_1+\vec{H}_2 = \frac{I}{2\pi b}\left[\frac{1-\cos\theta}{\sin\theta}\right]\vec{a}_z$$



$$\vec{H} = \vec{H}_0+\vec{H}_1+\vec{H}_2 = \frac{I}{2\pi b}\left[\pi-\theta+\frac{1-\cos\theta}{\sin\theta}\right]\vec{a}_z = 23.96\,\vec{a}_z\ \ A/m$$

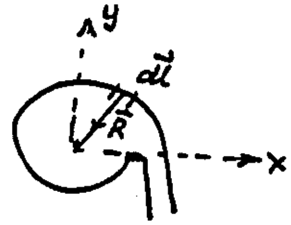
when $I = 10A,\ b = 0.2m,\ \theta = 15^\circ = 0.268$ rad.

$\vec{B} = \mu_0\vec{H} = 30.6\,\vec{a}_z\ \ \mu T$

Problem 5.24

$\vec{R} = -\rho\,\vec{a}_\rho = -a\,e^{-\phi/\pi}\,\vec{a}_\rho$

$d\vec{l} = \rho\,d\phi\,\vec{a}_\phi\ ,\quad d\vec{l}\times\vec{R} = a^2 e^{-2(\phi/\pi)}\,d\phi\,\vec{a}_z \Rightarrow$



$$H_z = \frac{I}{4\pi}\int_c \frac{d\vec{l}\times\vec{R}}{R^3} = \frac{I}{4\pi a}\int_0^{2\pi} e^{\phi/\pi}\,d\phi = \frac{I}{4a}\left(e^2-1\right)$$

When $I = 5A$, $a = 0.1\,m$, $\quad H_z = 79.86\ A/m$

Problem 5.25

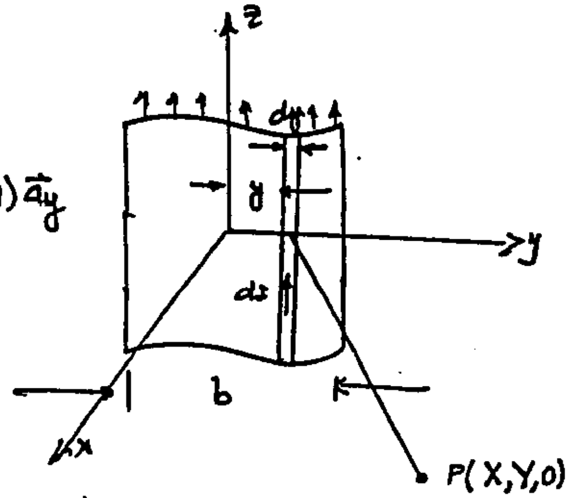
$\vec{J}_s = \frac{I}{b}\,\vec{a}_z\ ,\quad dI = \frac{I}{b}\,dy$

Using the result of thin-long wire carrying current

$\vec{R} = X\vec{a}_x + (Y-y)\vec{a}_y$

$d\vec{H} = \dfrac{I\,dy}{2\pi b R}\,\vec{a}_\phi$ where

$R = \sqrt{X^2 + (Y-y)^2}$ and



$\vec{a}_\phi = -\vec{a}_x \sin\phi + \vec{a}_y \cos\phi$

Thus,

$$\vec{H} = -\vec{a}_x\,\frac{I}{2\pi b}\int_{-b/2}^{b/2}\frac{(Y-y)\,dy}{(Y-y)^2+X^2} + \vec{a}_y\,\frac{I}{2\pi b}\int_{-b/2}^{b/2}\frac{X\,dy}{(Y-y)^2+X^2}$$

$$= \frac{I}{2\pi b}\left[\vec{a}_x\,\frac{1}{2}\ln\left(\frac{X^2+(Y-\frac{b}{2})^2}{X^2+(Y+\frac{b}{2})^2}\right) - \vec{a}_y\left\{\tan^{-1}\left(\frac{Y-b/2}{X}\right) - \tan^{-1}\left(\frac{Y+\frac{b}{2}}{X}\right)\right\}\right]$$

Problem 5.26

$m = NIA = 50\times 10\times 20\times 10^{-4} = 1$

Let $f = 3x + 4y + 12z - 26 =$ constant be the plane's surface, then

$\nabla f = 3\vec{a}_x + 4\vec{a}_y + 12\vec{a}_z \qquad |\nabla f| = 13$

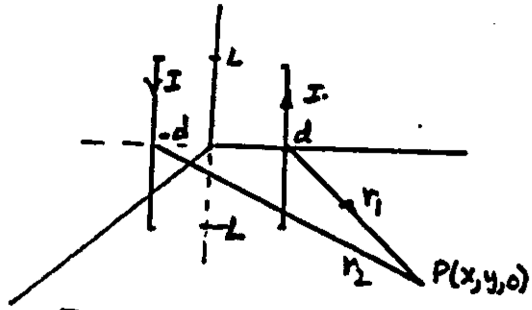
Thus. $\quad \vec{a}_n = \pm\dfrac{\nabla f}{|\nabla f|} = \pm\left[\frac{3}{13}\vec{a}_x + \frac{4}{13}\vec{a}_y + \frac{12}{13}\vec{a}_z\right]$

and $\quad \vec{m} = m\,\vec{a}_n = \pm\left[\frac{3}{13}\vec{a}_x + \frac{4}{13}\vec{a}_y + \frac{12}{13}\vec{a}_z\right]$

The + sign suggests that $\vec{m}$ is directed away from the origin.
Use the − sign when $\vec{m}$ is directed toward the origin.

Problem 5.27

$r_1 = \sqrt{(y-d)^2 + x^2} \qquad r_2 = \sqrt{(y+d)^2 + x^2}$



$A_{z1} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{2L}{r_1}\right) \;, \quad A_{z2} = -\frac{\mu_0 I}{2\pi} \ln\left[\frac{2L}{r_2}\right]$

$A_z = A_{z1} + A_{z2} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$

$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0 I}{2\pi}\left[\left(\frac{y+d}{r_2^2} - \frac{y-d}{r_1^2}\right)\vec{a}_x - \left(\frac{x}{r_2^2} - \frac{x}{r_1^2}\right)\vec{a}_y\right]$

$\mu_0 = 4\pi \times 10^{-7}$ H/m

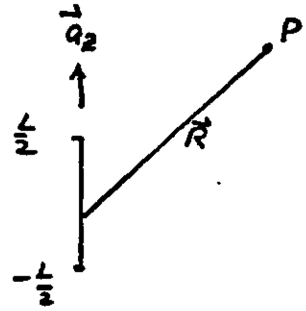
When $d = 1m$, $L = 5m$, $I = 10$ A, $x = 3m$, $y = 4m$,

$\vec{B} = -39.22\,\vec{a}_x + 156.86\,\vec{a}_y$ nT $\Rightarrow$ $\vec{H} = -31.21\,\vec{a}_x + 124.83\,\vec{a}_y$ A/m

Problem 5.28

$\vec{R} = r\vec{a}_r = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

$R = \sqrt{x^2 + y^2 + z^2} = r$



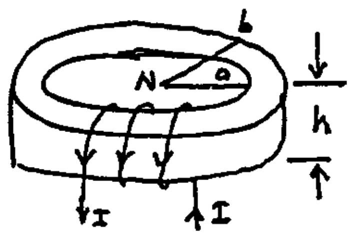
$A_z = \frac{\mu_0 I}{4\pi} \int_{-L/2}^{L/2} \frac{dz}{R}$, when $R \gg L \Rightarrow$

$A_z = \frac{\mu_0 I}{4\pi R} L$ and $\vec{B} = \nabla \times \vec{A} = \frac{\mu_0 IL}{4\pi r^2}\left[-\frac{y}{r}\vec{a}_x + \frac{x}{r}\vec{a}_y\right] = \frac{\mu_0 IL}{4\pi r^2} \sin\theta \, \vec{a}_\phi$

Problem 5.29

$a = 0.3m$, $b = 0.4m$, $h = 0.05m$

$N = 1000$ Turns, $I = 1A$, $\mu_r = 500$



$H_\phi = \frac{NI}{2\pi\rho} = \frac{159.155}{\rho}$ A/m

$\rho = 0.3$ m $\quad H_\phi$ (is max) $= 530.517$ A/m

$\rho = 0.4$ m $\quad H_\phi$ (min) $= 397.887$ A/m

$B_\phi = \mu_0 \mu_r H_\phi = 4\pi \times 10^{-7} \times 500 \times 159.155/\rho = \frac{0.1}{\rho}$ T $\qquad B_{max}\big|_{\rho=0.3} = 0.333$T

Thus. $\quad \Phi = \int_S \vec{B} \cdot \vec{ds} = \int_{0.3}^{0.4} \frac{0.1}{\rho} d\rho \int_0^{0.05} dz \qquad B_{min}\big|_{\rho=0.4} = 0.25$T

$= 0.1 \ln(0.4/0.3)(0.05) \cong 1.44$ mWb

Problem 5.30 $\quad I = 100\,A \quad a = 0.1\,m \quad \vec{J} = \frac{I}{\pi a^2}\vec{a}_z$

For $\rho \le a$: $\quad I_{enc} = \frac{I}{\pi a^2}\cdot \pi\rho^2 = I\left(\frac{\rho}{a}\right)^2 \Rightarrow H_\phi = \frac{I\rho}{2\pi a^2} \qquad \vec{J} = 3183.1\,\vec{a}_z\ A/m^2$

$\nabla\times\vec{H} = \frac{1}{\rho}\vec{a}_z\frac{\partial}{\partial\rho}\left(\frac{I\rho^2}{2\pi a^2}\right) = \frac{I}{\pi a^2}\vec{a}_z = \vec{J}$ as expected. $\qquad H_\phi = 1591.55\rho\ A/m$

For $\rho \ge a$: $\quad H_\phi = \frac{I}{2\pi\rho} = \frac{15.915}{\rho}\ A/m$ and $\nabla\times\vec{H} = 0$

Problem 5.31 $\quad \vec{J}_v = 200\,e^{-0.5\rho}\,\vec{a}_z\ A/m^2, \quad a = 0.1\,m$

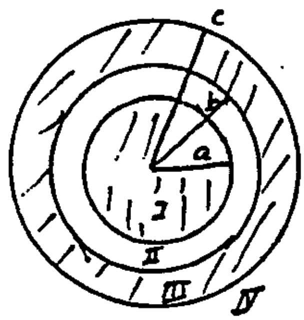
$\rho \le a$: $\quad I_{enc} = \int_s \vec{J}_v\cdot d\vec{s} = 200\int_0^\rho e^{-0.5\rho}\rho\,d\rho\int_0^{2\pi}d\phi = 400\pi\left[4 - 4e^{-0.5\rho} - 2\rho e^{-0.5\rho}\right]$

Thus, $\quad H_\phi = \frac{200}{\rho}\left[4 - 4e^{-0.5\rho} - 2\rho e^{-0.5\rho}\right]$

$\rho \ge a$: when $\rho = a$ yields: $\quad I_{enc} = 400\pi\left[4 - 4e^{-0.05} - 2(0.1)e^{-0.05}\right] = 6.078\,A$

and $\quad H_\phi = \frac{6.078}{2\pi\rho} = \frac{0.967}{\rho}\ A/m$

Problem 5.32



I: $\quad I_{enc} = \frac{I\rho^2}{a^2} \qquad H_\phi = \frac{I\rho}{2\pi a^2} \qquad \rho \le a$

II: $\quad a \le \rho \le b$: $\quad H_\phi = \frac{I}{2\pi\rho}$

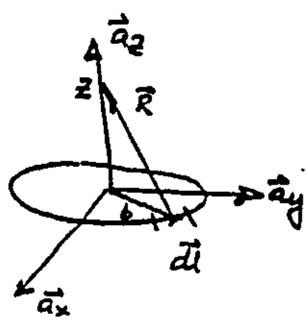
III: $\quad b \le \rho \le c$: $\quad I_{enc} = I - I\frac{\rho^2 - b^2}{c^2 - b^2} = I\frac{c^2-\rho^2}{c^2-b^2} \Rightarrow H_\phi = \frac{I}{2\pi\rho}\left[\frac{c^2-\rho^2}{c^2-b^2}\right]$

IV: $\quad I_{enc} = 0 \qquad H_\phi = 0$

Problem 5.33 $\quad \Phi = \int_s \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int_a^b \frac{1}{\rho}d\rho\int_0^1 dz = \frac{\mu_0 I}{2\pi}\ln(b/a)\ \ Wb/m$

$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{\mu_0}{8\pi^2}\cdot\frac{I^2}{\rho^2} \Rightarrow W_m = \frac{\mu_0}{8\pi^2}\int_a^b\frac{1}{\rho}d\rho\int_0^{2\pi}d\phi\int_0^1 dz = \frac{\mu_0 I^2}{4\pi}\ln(b/a)\ \ J/m.$

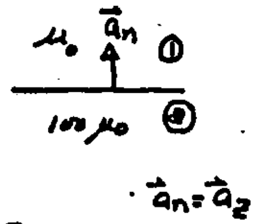
Problem 5.34 $\quad d\vec{l} = b\,d\phi\,\vec{a}_\phi, \ \vec{R} = -b\vec{a}_\rho + z\vec{a}_z$



$\vec{H} = \frac{I}{4\pi}\int_c\frac{d\vec{l}\times\vec{R}}{R^3} = \frac{Ib^2}{4\pi}\int_0^{2\pi}\frac{d\phi\,\vec{a}_z}{(b^2+z^2)^{3/2}} = \frac{Ib^2}{2(b^2+z^2)^{3/2}}\vec{a}_z$

$\int_c \vec{H}\cdot d\vec{l} = \int_{-\infty}^{\infty}H_z\,dz = \frac{Ib^2}{2}\int_{-\infty}^{\infty}\frac{dz}{(b^2+z^2)^{3/2}} = \frac{I}{2}\left[\frac{z}{\sqrt{b^2+z^2}}\right]_{-\infty}^{\infty} = I$

Problem 5.35 $\vec{B}_1 = 1.5\vec{a}_x + 0.8\vec{a}_y + 0.6\vec{a}_z$ mT



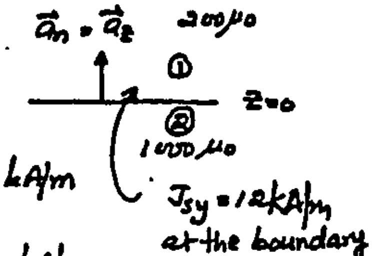
$B_{n1} = B_{n2} \Rightarrow B_{z2} = B_{z1} = 0.6$ mT

$\vec{a}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \Rightarrow \vec{a}_z \times \left(\frac{\vec{B}_1}{\mu_0} - \frac{\vec{B}_2}{100\mu_0}\right) = 0 \quad \because \vec{J}_s = 0$

or $B_{x2} = 100\,B_{x1} = 150$ mT and $B_{y2} = 100\,B_{y1} = 80$ mT

$$\vec{B}_2 = 150\,\vec{a}_x + 80\,\vec{a}_y + 0.6\,\vec{a}_z \text{ mT}$$

Problem 5.36



$\vec{H}_1 = 40\,\vec{a}_x + 50\,\vec{a}_y + 12\,\vec{a}_z$ kA/m, $\vec{B}_1 = \mu_1\vec{H}_1 = 200\mu_0\vec{H}_1$

$B_{z2} = B_{z1} \Rightarrow 200\mu_0(12\times 10^3) = 1000\mu_0 H_{z2}$ or $H_{z2} = 2.4$ kA/m

$\vec{a}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \Rightarrow H_{x1} - H_{x2} = J_{sy} \Rightarrow H_{x2} = 40 - 12 = 28$ kA/m

$H_{y1} - H_{y2} = 0 \Rightarrow H_{y2} = 50$ kA/m. $\vec{H}_2 = 28\,\vec{a}_x + 50\,\vec{a}_y + 2.4\,\vec{a}_z$ kA/m

Problem 5.37 Region-1: Free space $\chi_m = \mu_r - 1 = 0$, $\vec{J}_{sv} = 0$, $\vec{J}_{sb} = 0$

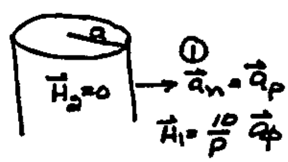
Region-2: $\chi_m = 100 - 1 = 99$, $\vec{M} = \chi_m \vec{H}_2 = \frac{\chi_m}{100\mu_0}\vec{B}_2 = \frac{0.99}{\mu_0}(150\,\vec{a}_x + 80\,\vec{a}_y + 0.6\,\vec{a}_z)$ mT

$\vec{J}_{sv} = \nabla \times \vec{M} = 0$

At the boundary: $\vec{J}_{sb} = \vec{M} \times (-\vec{a}_z) = 118.2\,\vec{a}_y - 63.03\,\vec{a}_x$ kA/m

Problem 5.38 $\vec{a}_n \times (\vec{H}_1 - \overset{0}{\cancel{\vec{H}_2}}) = \vec{J}_s \Rightarrow \vec{J} = \frac{10}{a}\vec{a}_z$ A/m

(at $\rho = a$) $a = 0.1$ m



Thus, $\vec{J}_s = 100\,\vec{a}_z$ A/m and

$I = 2\pi \times 0.1 \times 100 = 62.83$ A

Problem 5.39 $\mu_{r1} = \frac{1.2}{4\pi\times 10^{-7}\times 300} \cong 3183$ $\quad \mu_{r2} = \frac{1.5}{4\pi\times 10^{-7}\times 1500} = 795.78$

$M_1 = 3182 \times 300 = 954.6$ kA/m, $M_2 = 794.78 \times 1500 = 1.19\times 10^6$ A/m

$$\%\ \text{Increase} = \left(\frac{M_2 - M_1}{M_1}\right)100 = 24.89\%$$

Problem 5.40

Path	l (cm)	A (cm^2)	$R = l/\mu A$ (H^{-1})
ab	36	4	358.099×10^3
bc	56	8	278.521×10^3
cd	36	12	119.366×10^3
da	56	8	278.521×10^3

$\Sigma = 1.0345 \times 10^6 = R_T$

$\Phi = \dfrac{NI}{R_T} = \dfrac{200 \times 10^{-6}}{1.0345} = 193.3\ \mu Wb$, $\mu = 2000\mu_0 = 2.513 \times 10^{-3}$ H/m

Problem 5.41

Path	Area (cm^2)	l (cm)	B(T)	w_m (J/m^3)	W_m (mJ)
ab	4	36	0.483	46.416	6.684
bc	8	56	0.242	11.652	5.22
cd	12	36	0.161	5.157	2.228
da	8	56	0.242	11.652	5.22

Total Energy = 19.352 mJ

Since $I = 0.2$ A $\Rightarrow L = 2 \times 19.352 \times 10^{-3}/0.2^2 \cong 0.97$ H

Problem 5.42 $L = \dfrac{N^2}{R_T} = \dfrac{1000^2}{1.0345} \times 10^{-6} \cong 0.97\,H$, $W_m = \frac{1}{2} \times 0.97 \times 0.2^2 \cong 19.4$ mJ

Problem 5.43 $\mu = 4\pi \times 10^{-7} \times 500$

Path	l (cm)	A (cm^2)	R (H^{-1})
ab	0.05	12	331,573
bc	10	12	132,629
cd	8	24	53,052
de	16	12	212,207
ef	8	24	53,052
fa	5.95	12	78,914
R_T	=		861,427

$\Phi = 1.44$ mWb

mmf: $\Rightarrow \mathcal{F} = \Phi R_T = 1240.45$ At

mmf supplied by 500-coil = 500×0.8 = 400

mmf to be supplied by the 700-coil: = 1240.45 − 400 = 840.45

Ans: $I_{700} = \dfrac{840.45}{700} \cong 1.2$ A

Problem 5.44

Path	Φ(mWb)	A(cm²)	B(T)	H(A/m)	l(cm)	Hl
ab	1.44	12	1.2	954,930	0.05	477.46
bc	1.44	12	1.2	1300	10	130.00
cd	1.44	24	0.6	600	8	48.00
de	1.44	12	1.2	1300	16	208.00
ef	1.44	24	0.6	600	8	48.00
fa	1.44	12	1.2	1300	5.95	77.35

Total mmf drop = 988.81

mmf supplied by 550-coil = 400 $\Rightarrow$ $I_{700} = \dfrac{988.81-400}{700} = 0.84\,A$

Problem 5.45

Path	lA(cm³)	B(T)	$w_m = \frac{1}{2}\frac{B^2}{\mu}$	$W_m = w_m lA$
ab	0.6	1.2	572,958	0.344
bc	120	1.2	1145.91	0.138
cd	192	0.6	286.48	0.055
de	192	1.2	1145.91	0.220
ef	192	0.6	286.48	0.055
fa	71.4	1.2	1145.91	0.082
			W_{mT} =	0.894

From Problem 5.43,

equivalent $I = \dfrac{1240.45}{1200} = 1.0337\,A$

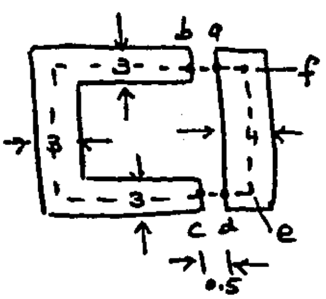
Since $W_{mT} = \frac{1}{2}LI^2 \Rightarrow$

$$L = \frac{2\times 0.894}{(1.0337)^2} = 1.67\,H$$

Since, $\mathcal{R}_T = 861,427\ H^{-1}$

$$L = \frac{N^2}{\mathcal{R}_T} = \frac{1200^2}{861,427} = 1.67\,H$$

Problem 5.46



$N = 1600 \Rightarrow$

$$I = \frac{6414.82}{1600} = 4.01\,A$$

Path	Φ(mWb)	A(cm²)	B(T)	H(A/m)	l(cm)	Hl
ab	1.125	15	0.75	596,831	0.5	2984.16
bc	1.125	15	0.75	650	52	338
cd	1.125	15	0.75	596,831	0.5	2984.16
de	1.125	15	0.75	650	2	13
ef	1.125	20	0.56	550	15	82.5
fa	1.125	15	0.75	650	2	13

mmf = 6414.82

Problem 5.47   50% increase in mmf ⇒ $\mathcal{F} = 1.5 \times 6414.82 \cong 9622$ At

mmf drop across the air-gap without the increase is 93%. Let us assume the increased mmf drop across each air-gap is 91% ⇒ ≅ 4400 At.

First Iteration

Path	Flux (mWb)	Area (cm^2)	B(T)	H (A/m)	l (cm)	Hl
ab	1.659	15	1.106	880,000	0.5	4400
bc	"	15	1.106	1,000	52	520
cd	"	15	1.106	880,000	0.5	4400
de	"	15	1.106	1,000	2	20
ef	"	20	0.83	700	15	105
fa	"	15	1.106	1,000	2	20
					Σ	9465

% Error: $\frac{9600 - 9465}{9600} \times 100 = 1.4\ \%$

No further iteration is necessary

Problem 5.48

Path	A (cm^2)	l (cm)	B(T)	$w_m = \frac{1}{2}\frac{B^2}{\mu}$	$W_m = w_m A l$
ab	15	0.5	0.75	223,812	1.68
bc	15	52	0.75	243.75	0.19
cd	15	0.5	0.75	223,812	1.68
de	15	2	0.75	243.75	0.01
ef	20	15	0.56	154,00	0.05
fa	15	2	0.75	243.75	0.01

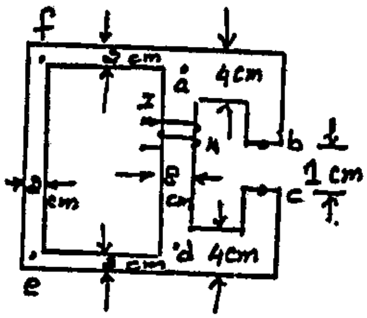
$W_{mT} = \Sigma = 3.62\ J$

$$L = \frac{2 W_{mT}}{I^2} = \frac{2 \times 3.62}{4.01^2} = 0.45\ H$$

$$\mathcal{R} = \frac{mmf}{\Phi} = \frac{6414.82}{1.125 \times 10^{-3}} = 5.702 \times 10^{6}\ H^{-1}$$

$$L = \frac{N^2}{\mathcal{R}} = \frac{1600^2}{5.702} 10^{-6} = 0.45\ H$$

Problem 5.49



N = 200 Turns

Path	Flux (mWb)	A (cm^2)	B(T)	H(A/m)	ℓ(cm)	$H\ell$
ab	0.64	32	0.2	250	26.5	66.3
bc	0.64	32	0.2	159155	1	1591.6
cd	0.64	32	0.2	250	26.5	66.3
mmf drop across abcd						1724.2
afed	1.064	8	1.33	1582	109	1724.2
ad	1.704	96	0.18	220	19	43.7
						1767.9

Thus $I = 1767.9/200 = 8.84A$

$\mathcal{R} = \frac{1767.9}{1.704} \times 10^3 = 1.038 \times 10^6\ H$ $\quad L = N^2/\mathcal{R} = 38.54$ mH $\quad W_m = \frac{1}{2}LI^2 = 1.51$ J

Problem 5.50 20% increase in the current ⇒ 20% increase in mmf.

New mmf = $1767.9 \times 1.2 = 2121.48$ A-t

Expected new mmf drop across air-gap: $\frac{1591.6}{1767.9} \times 2121.48 \cong 1910$

First-Iteration

Path	Flux (mWb)	A (cm^2)	B(T)	H(A/m)	ℓ(cm)	$H\ell$ (At)
ab	0.768	32	0.24	290	26.5	77
bc	0.768	32	0.24	191,000	1	1910
cd	0.768	32	0.24	290	26.5	77
						2064
afed	1.152	8	1.44	1894	109	2064
ad	1.92	96	0.2	250	19	48
						$\Sigma \cong 2112$

% Error: $\frac{2121.48 - 2112}{2121.48} \times 100 \cong 0.45\%$

Flux density in air-gap: $B_g = 0.24$ T

No further iteration is necessary.